Ahmad Basyir Najwan - 1906285314

Tugas Integral Lipat 3

$\boxed{\text{Subbab 13.7}}$

contoh 1 :

$$= \iiint_B x^2yz \, dx\,dy\,dz \qquad B = \{(x,y,z) : 1 \le x \le 2, 0 \le y \le 1, 0 \le z \le 6\}$$

$$= \int_1^2 \int_0^1 \int_0^6 x^2yz \, dz\,dy\,dx$$

$$= \int_1^2 x^2\,dx \int_0^1 y\,dy \int_0^6 z\,dz$$

$$= \left(\frac{x^3}{3}\right)_1^2 \left(\frac{y^2}{2}\right)_0^1 \left(\frac{z^2}{2}\right)_0^6$$

$$= \left(\frac{8}{3} - \frac{1}{3}\right)\left(\frac{1}{2} - 0\right)\left(\frac{36}{2} - 0\right)$$

$$= \left(\frac{7}{3}\right)\left(\frac{1}{2}\right)(18)$$

$$= 21$$

$$\boxed{\iiint_B x^2yz \, dx\,dy\,dz = 21}$$

contoh 2 :

① $$\int_0^2 \int_0^{3x} \int_{y+1}^{2x} 6 \, dz\,dy\,dx = \int_0^2 \int_0^{3x} \left[6z\right]_{y+1}^{2x} dy\,dx$$

$$= \int_0^2 \int_0^{3x} (12x - 6y - 6)\,dy\,dx$$

$$= \int_0^2 \left[12xy - 3y^2 - 6y\right]_0^{3x} dx$$

$$= \int_0^2 \left[36x^2 - 27x^2 - 18x - 0\right] dx$$

$$= \int_0^2 (9x^2 - 18x)\,dx$$

$$= \left[3x^3 - 9x^2\right]_0^2$$

$$= (24 - 36 - 0) = -12$$

$$\boxed{\int_0^2 \int_0^{3x} \int_{y+1}^{2x} 6 \, dz\,dy\,dx = -12}$$

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contoh 2 :

③ $$\int_0^{\pi/2}\int_0^{z}\int_0^{y} \sin(x+y+z)\,dz\,dy\,dx = \int_0^{\pi/2}\int_0^{z}\int_0^{y} \sin(x+y+z)\,dx\,dy\,dz$$

$$= \int_0^{\pi/2}\int_0^{z} \Big[-\cos(x+y+z)\Big]_0^{y} dy\,dz$$

$$= \int_0^{\pi/2}\int_0^{z} \Big[\cos(y+z) - \cos(2y+z)\Big] dy\,dz$$

$$= \int_0^{\pi/2} \Big[\sin(y+z) - \tfrac{1}{2}\sin(2y+z)\Big]_0^{z} dz$$

$$= \int_0^{\pi/2} \Big[\sin(2z) - \tfrac{1}{2}\sin(3z) - \sin(z) + \tfrac{1}{2}\sin(z)\Big] dz$$

$$= \int_0^{\pi/2} \Big[\sin(2z) - \tfrac{1}{2}\sin(3z) - \tfrac{1}{2}\sin(z)\Big] dz$$

$$= \Big[-\tfrac{1}{2}\cos(2z) + \tfrac{1}{6}\cos(3z) + \tfrac{1}{2}\cos(z)\Big]_0^{\pi/2} dz$$

$$= \Big[-\tfrac{1}{2}\cos(\pi) + \tfrac{1}{6}\cos\Big(\tfrac{3\pi}{2}\Big) + \tfrac{1}{2}\cos\Big(\tfrac{\pi}{2}\Big) - \Big(-\tfrac{1}{2}\cos(0) + \tfrac{1}{6}\cos(0) + \tfrac{1}{2}\cos(0)\Big)\Big]$$

$$= \frac{1}{2} - \Big(-\frac{1}{2} + \frac{1}{6} + \frac{1}{2}\Big)$$

$$= \frac{1}{3}$$

$$\boxed{\int_0^{\pi/2}\int_0^{z}\int_0^{y} \sin(x+y+z)\,dx\,dy\,dz = \frac{1}{3}}$$

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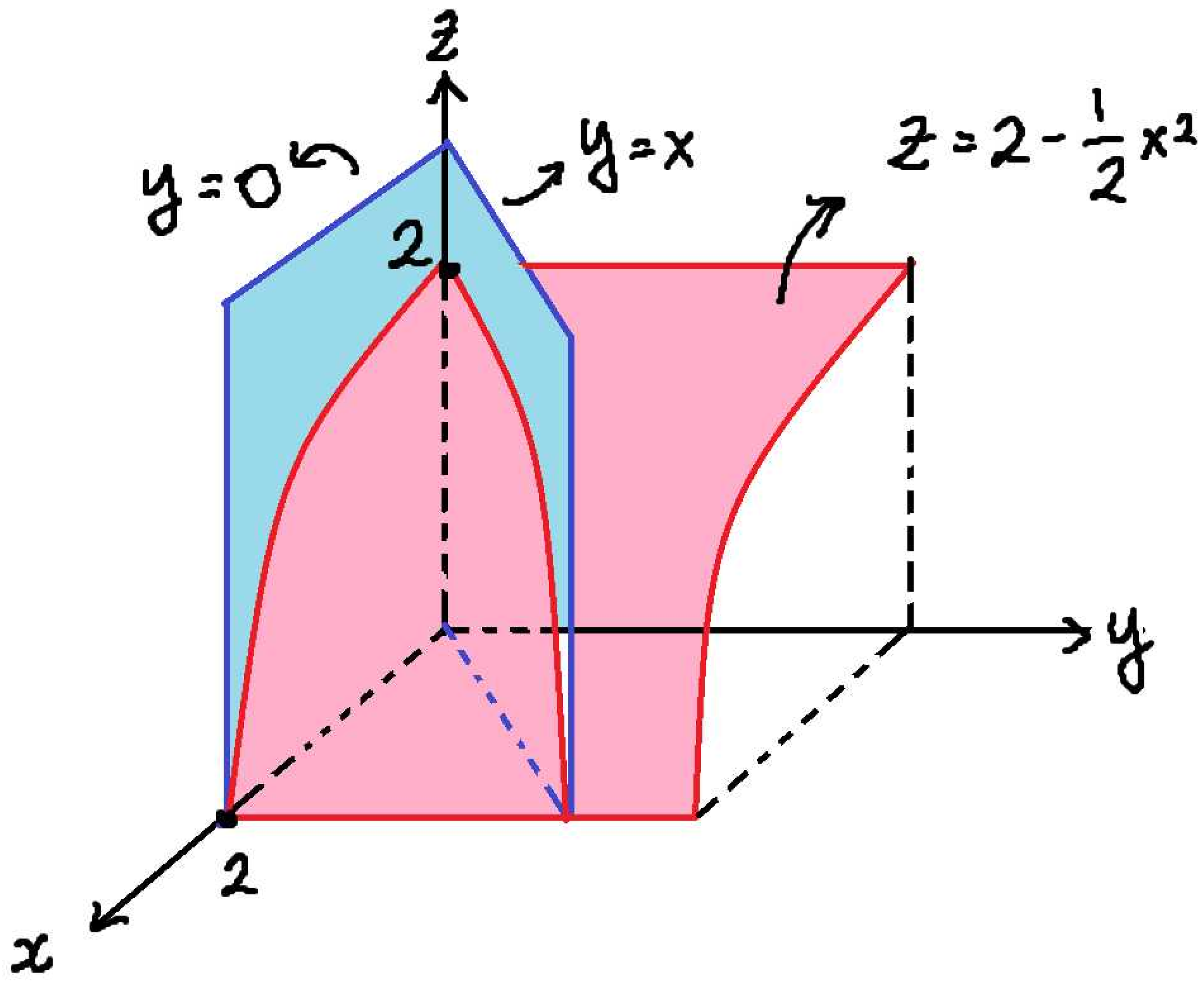
Tugas Integral Lipat 3

$\boxed{\text{Subbab 13.7}}$

contoh 3:

$f(x,y,z) = 2xyz$

$$S = \begin{cases} z = 2 - \frac{1}{2}x^2 \\ x = y,\ y = 0,\ z = 0 \end{cases}$$



$z \Rightarrow 0 \leq z \leq 2 - \frac{x^2}{2}$

$y \Rightarrow 0 \leq y \leq x$

$x \Rightarrow 0 \leq x \leq 2$

$$\iiint_S f(x,y,z)\,dV = \int_0^2 \int_0^x \int_0^{2-\frac{x^2}{2}} 2xyz\ dz\,dy\,dx$$

$$= 2\int_0^2 \int_0^x \int_0^{2-\frac{x^2}{2}} z\,dz\ y\,dy\ x\,dx$$

$$= 2\int_0^2 \int_0^x \left[\frac{z^2}{2}\right]_0^{2-\frac{x^2}{2}} y\,dy\ x\,dx$$

$$= \frac{1}{4}\int_0^2 \int_0^x (4-x^2)^2 y\,dy\ x\,dx$$

$$= \frac{1}{4}\int_0^2 \left[(4-x^2)^2 \frac{y^2}{2}\right]_0^x x\,dx$$

$$= \frac{1}{8}\int_0^2 (4-x^2)^2 x^3\,dx$$

$$= \frac{1}{8}\int_0^2 (16x^3 - 8x^5 + x^7)\,dx$$

$$= \frac{1}{8}\left[4x^4 - \frac{4}{3}x^6 + \frac{1}{8}x^8\right]_0^2$$

$$= \frac{1}{8}\left(64 - \frac{256}{3} + 32\right)$$

$$= 12 - \frac{32}{3}$$

$$= \frac{4}{3} \approx 1{,}333$$

$$\boxed{V = \frac{4}{3} \approx 1{,}333}$$

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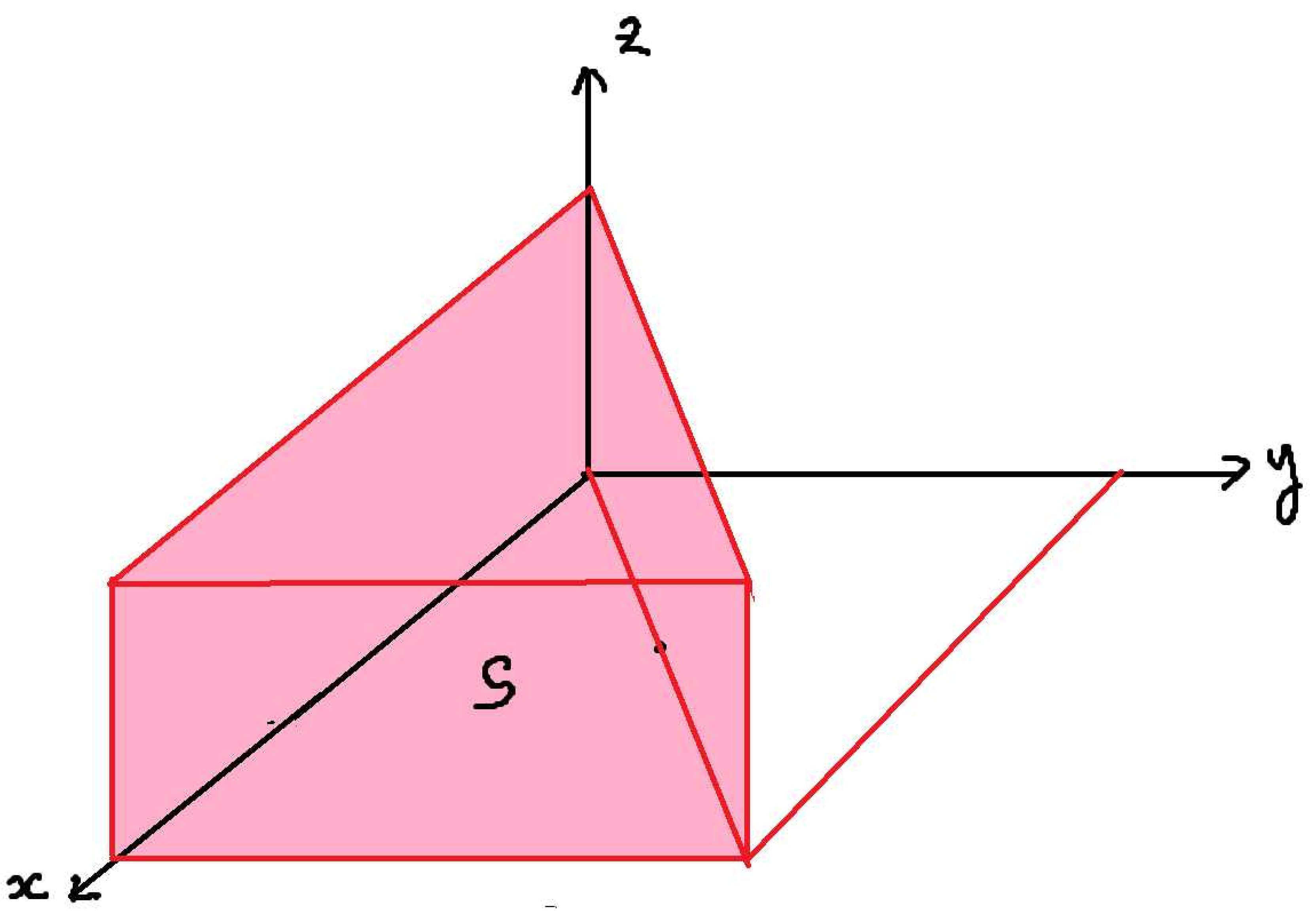
Tugas Integral Lipat 3

Subbab 13.7

Latihan

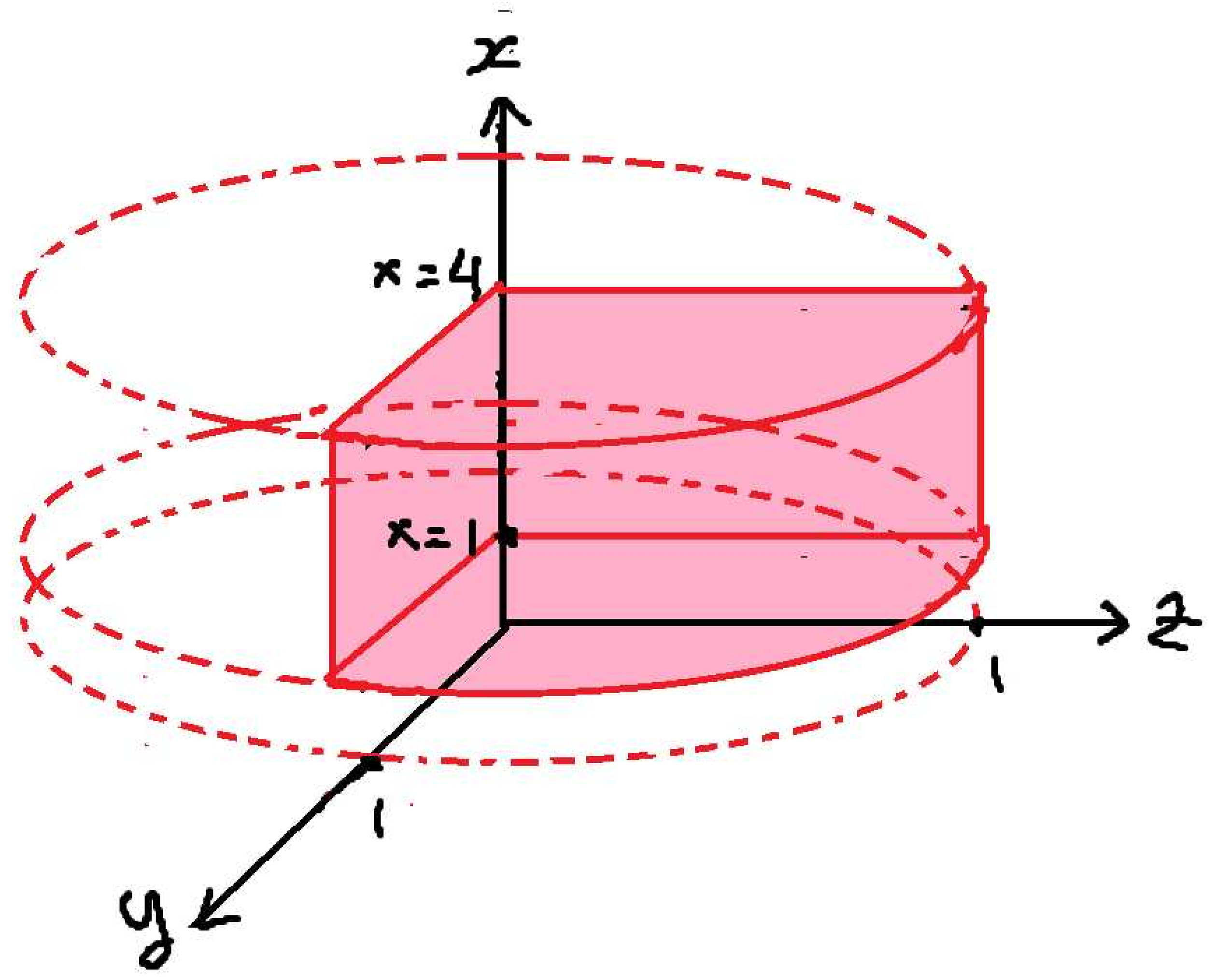
$$\iiint_S f(x,y,z)\,dV$$

① $S = \{(x,y,z) : 0 \leq x \leq y/2,\ 0 < y < 4,\ 0 \leq z \leq 2$



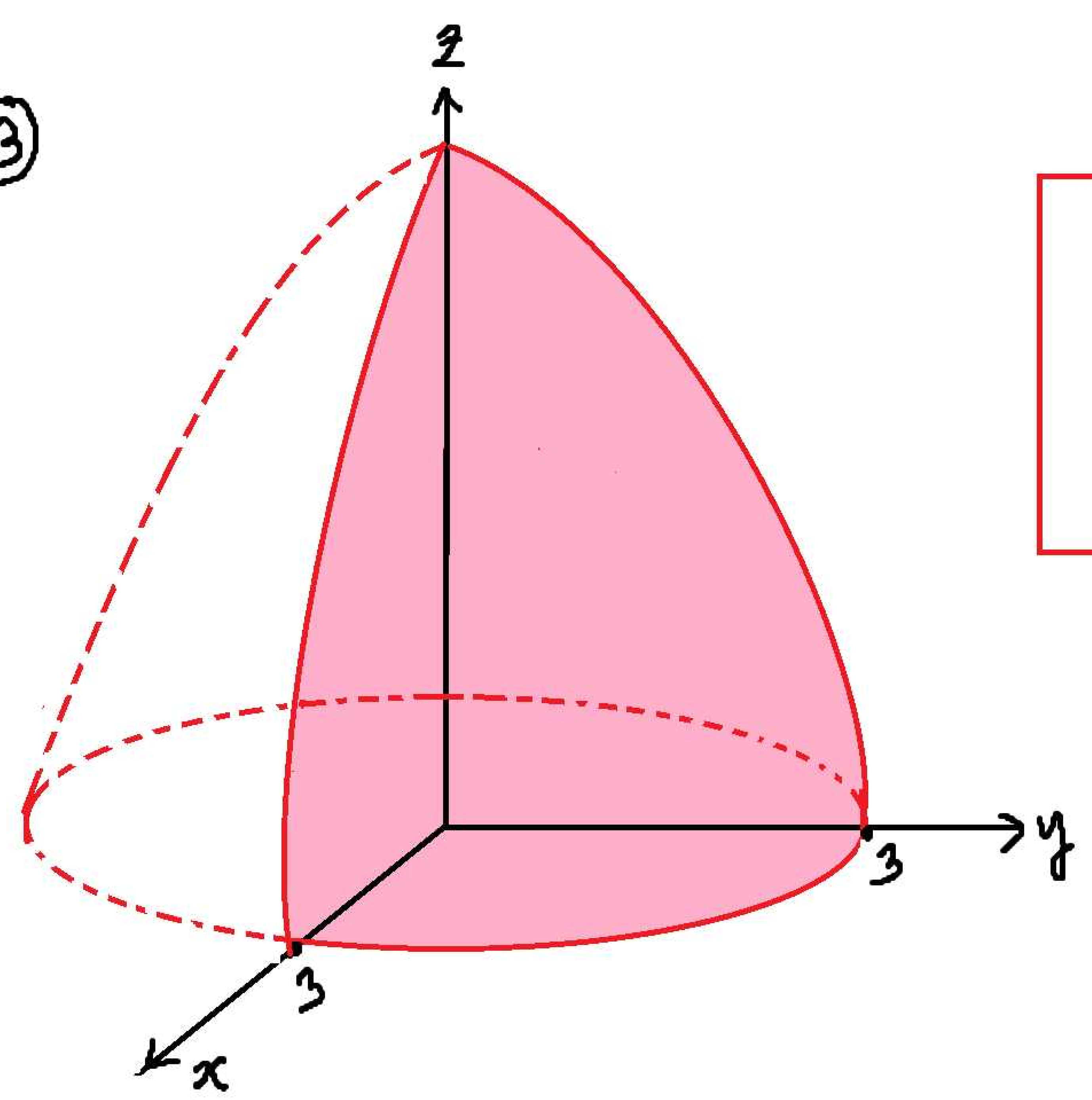
$$\boxed{\int_0^2 \int_0^4 \int_0^{y/2} f(x,y,z)\ dx\,dy\,dz}$$

②



$$\boxed{\int_1^4 \int_0^1 \int_0^{\sqrt{1-y^2}} f(x,y,z)\ dz\,dy\,dx}$$

③



$$\boxed{\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{9-x^2-y^2} f(x,y,z)\ dz\,dy\,dx}$$

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Tugas Integral Lipat 3

$\boxed{\text{Subbab 13.8}}$

Contoh 1:

① $$\int_0^{\pi}\int_0^{3}\int_0^{6} r\,dz\,dr\,d\theta = \int_0^{\pi} d\theta \int_0^{3} r\,dr \int_0^{6} dz$$

$$= (\theta)_0^{\pi}\left(\frac{r^2}{2}\right)_0^3 (z)_0^6$$

$$= (\pi)\left(\frac{9}{2}\right)(6)$$

$$= 27\pi$$

$$\boxed{\int_0^{\pi}\int_0^{3}\int_0^{6} r\,dz\,dr\,d\theta = 27\pi}$$

② $$\int_0^{\pi/2}\int_0^{2}\int_0^{4-r^2} zr\,dz\,dr\,d\theta = \int_0^{\pi/2} d\theta \int_0^{2}\left(\int_0^{4-r^2} z\,dz\right) r\,dr$$

$$= [\theta]_0^{\pi/2}\int_0^2 \left[\frac{z^2}{2}\right]_0^{4-r^2} r\,dr$$

$$= \left(\frac{\pi}{2}\right)\int_0^2 \left(\frac{1}{2}(4-r^2)^2\right) r\,dr$$

$$= -\frac{\pi}{8}\int_0^2 (4-r^2)^2\,d(4-r^2)$$

$$= -\frac{\pi}{8}\left[\frac{1}{3}(4-r^2)^3\right]_0^2$$

$$= -\frac{\pi}{8}\left[0-\left(\frac{64}{3}\right)\right]$$

$$= \frac{8\pi}{3}$$

$$\boxed{\int_0^{\pi/2}\int_0^{2}\int_0^{4-r^2} zr\,dz\,dr\,d\theta = \frac{8\pi}{3}}$$

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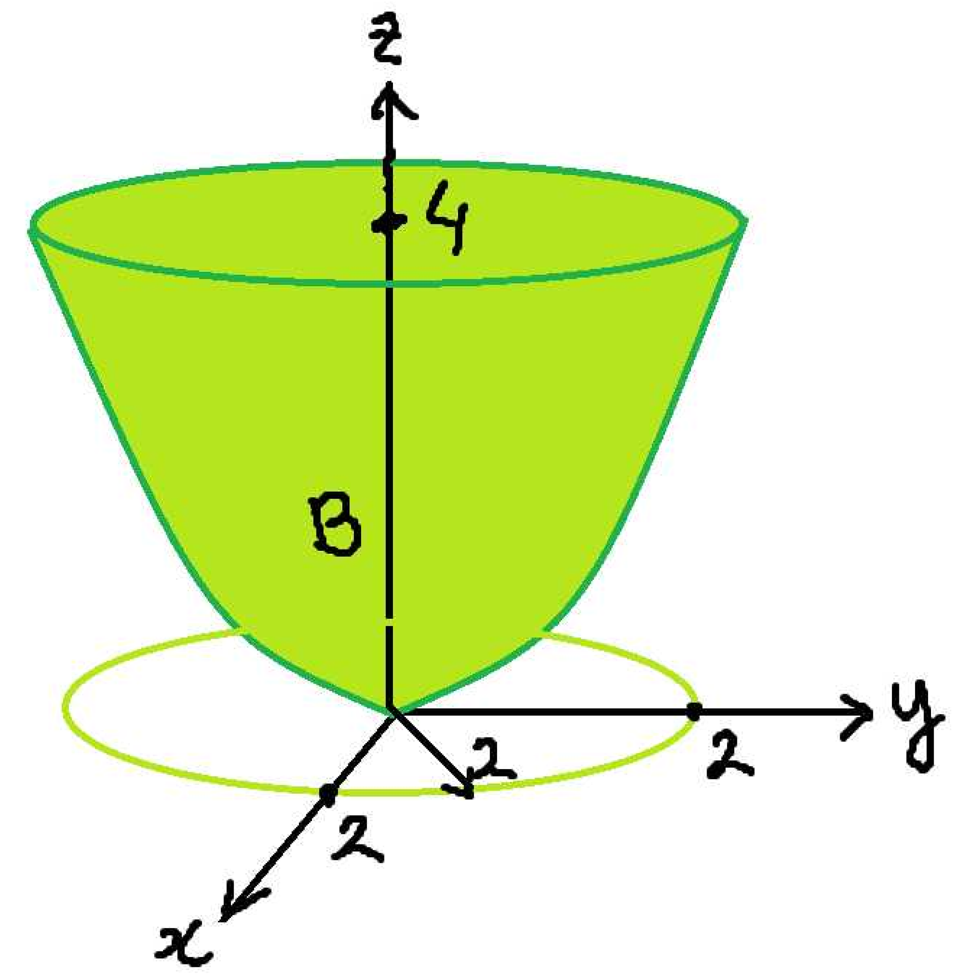
Tugas Integral Lipat 3

$\boxed{\text{Subbab 13.8}}$

contoh 2:

$z = x^2 + y^2 \qquad z = 4$



$z \Rightarrow \quad x^2+y^2 \leq z \leq 4$

$y \Rightarrow \quad -\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2}$

$x \Rightarrow \quad -2 \leq x \leq 2$

$$V = \iiint_B dz\,dy\,dx$$

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} dz\,dy\,dx$$

$$\boxed{\int_{-a}^{a}(\text{fungsi genap})\,dx = 2\int_{0}^{a}(\text{fungsi genap})\,dx}$$

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\Big[z\Big]_{x^2+y^2}^{4} dy\,dx$$

$f(x,y) = 4-x^2-y^2 \Rightarrow$ fungsi genap

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\left(4-x^2-y^2\right)dy\,dx = 2\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\left(4-x^2-y^2\right)dy\,dx$$

$$= 2\int_{-2}^{2}\left[4y - x^2y - \frac{1}{3}y^3\right]_{0}^{\sqrt{4-x^2}} dx$$

$$= 2\int_{-2}^{2}\left[4\sqrt{4-x^2} - x^2\sqrt{4-x^2} - \frac{1}{3}(4-x^2)^{\frac{3}{2}}\right]dx$$

$$= 2\int_{-2}^{2}\left[(4-x^2)\sqrt{4-x^2} - \frac{1}{3}(4-x^2)^{\frac{3}{2}}\right]dx$$

$f(x) = (4-x^2)^{\frac{3}{2}} \Rightarrow$ fungsi genap

$$= 2\int_{-2}^{2}\frac{2}{3}(4-x^2)^{\frac{3}{2}}dx = \frac{4}{3}\int_{-2}^{2}(4-x^2)^{\frac{3}{2}}dx$$

$$= \frac{8}{3}\int_{0}^{2}(4-x^2)^{\frac{3}{2}}dx = \frac{8}{3}\left(6\sin^{-1}\left(\frac{x}{2}\right) - \frac{1}{4}x\sqrt{4-x^2}\,(x^2-10)\right)_{0}^{2}$$

$$= \frac{8}{3}\left(6\sin^{-1}(1)\right) = \frac{8}{3}\left(6\,\frac{\pi}{2}\right) = 8\pi = 25{,}1327$$

$$\boxed{V = 8\pi = 25{,}1327}$$

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Tugas Integral Lipat 3

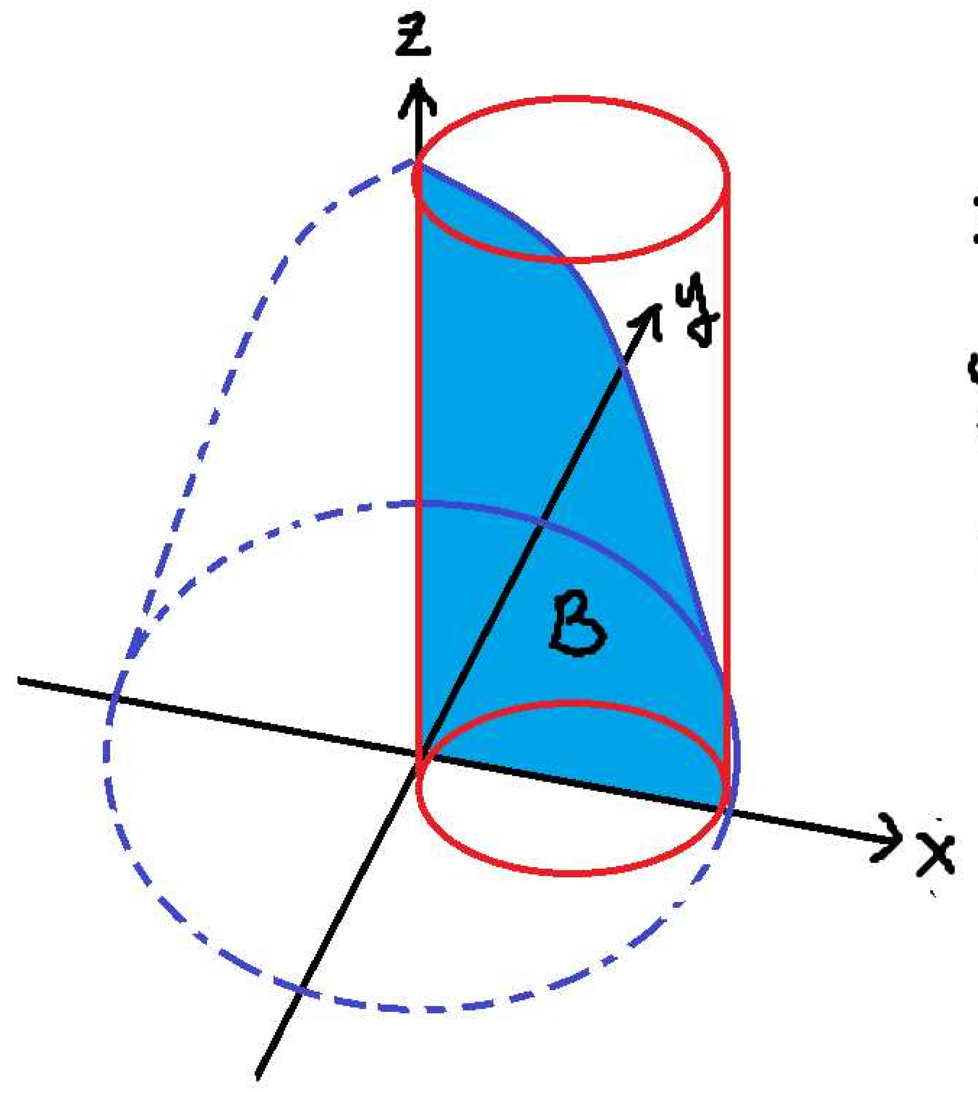
$\boxed{\text{Subbab 13.8}}$

Contoh 3:

$$\left.\begin{array}{l} z = 4 - x^2 - y^2 \\ x^2 + y^2 = 2x \end{array}\right\} \text{di Okton Pertama}$$

$$\downarrow$$

$$(x-1)^2 + y^2 = 1$$



$z \Rightarrow 0 \leq z \leq 4 - x^2 - y^2$

$y \Rightarrow 0 \leq y \leq \sqrt{2x - x^2}$

$x \Rightarrow 0 \leq x \leq 2$

$$V = \iiint_B dz\,dy\,dx$$

$$= \int_0^2 \int_0^{\sqrt{2x-x^2}} \int_0^{4-x^2-y^2} dz\,dy\,dx$$

$$= \int_0^2 \int_0^{\sqrt{2x-x^2}} \left[z\right]_0^{4-x^2-y^2} dy\,dx$$

$$= \int_0^2 \int_0^{\sqrt{2x-x^2}} (4 - x^2 - y^2)\,dy\,dx$$

$$= \int_0^2 \left[(4-x^2)y - \frac{1}{3}y^3\right]_0^{\sqrt{2x-x^2}} dx$$

$$= \int_0^2 \left[\left((4-x^2) - \frac{1}{3}(2x-x^2)\right)\sqrt{2x-x^2}\right] dx$$

$$= \frac{2}{3}\int_0^2 \left(6 - x^2 - x\right)\sqrt{2x-x^2}\;dx$$

$$= \frac{2}{3}\left[-\frac{\sqrt{2x-x^2}\left(\sqrt{x^2-2x}\,(2x^3+2x^2-27x+15) + 30\ln\left(\sqrt{x-2}+\sqrt{x}\right)\right)}{8\sqrt{x^2-2x}}\right]_0^2$$

$$= \frac{2}{3}\left[\frac{15\pi}{8}\right] = \frac{5\pi}{4} = 3{,}92699$$

$$\boxed{V = \frac{5\pi}{4} = 3{,}92699}$$