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PR 3 Fismat : Persamaan Diferensial Biasa

1) $y'' = y$

$y = \cosh x$
$y' = \sinh x$
$y'' = \cosh x$
$\Downarrow$
$y'' = y$ //

$y = \sinh x$
$y' = \cosh x$
$y'' = \sinh x$
$\Downarrow$
$y'' = y$ //

2) (i) $f = -kv$

$$\Sigma F = ma$$
$$mg - kv = m\frac{dv}{dt}$$
$$\frac{dv}{dt} + \frac{k}{m}v = g \quad \times e^{\frac{k}{m}t}$$
$$e^{\frac{k}{m}t}\frac{dv}{dt} + e^{\frac{k}{m}t}\frac{k}{m}v = e^{\frac{k}{m}t}g$$
$$\frac{d}{dt}\left(e^{\frac{k}{m}t}v\right) = e^{\frac{k}{m}t}g$$
$$e^{\frac{k}{m}t}v = g\int e^{\frac{k}{m}t} + C$$

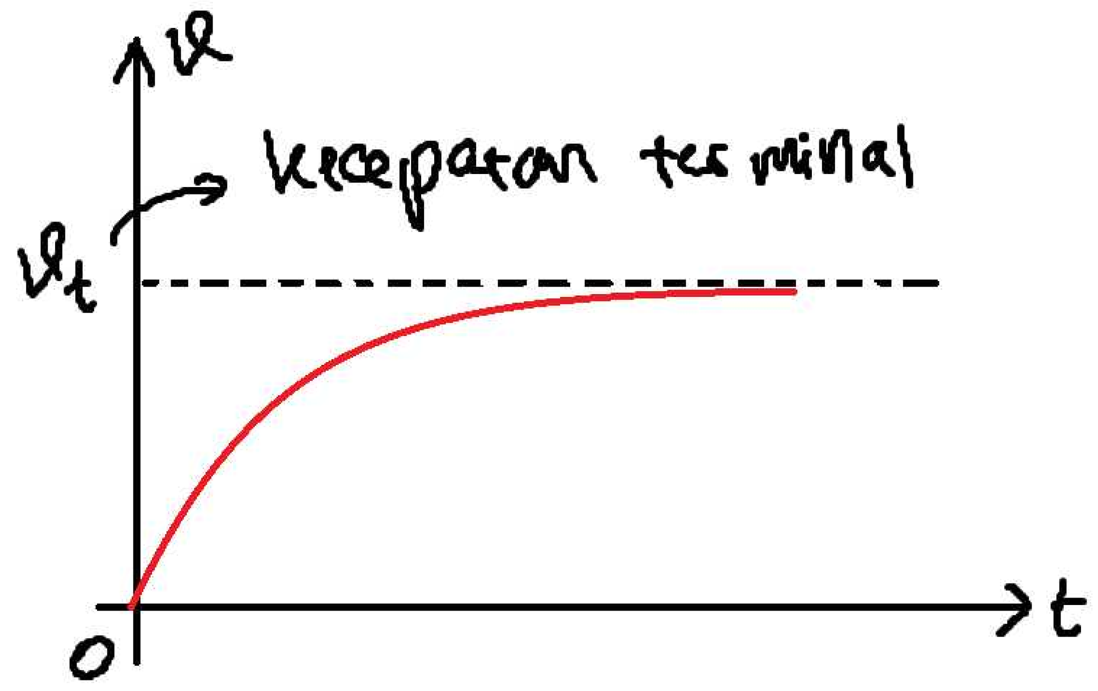
$$v(t) = \frac{mg}{k} + C\cdot e^{-\frac{k}{m}t}$$
$$v(0) = 0$$
$$\frac{mg}{k} + C\underbrace{e^{0}}_{=1} = 0$$
$$C = -\frac{mg}{k}$$
$$v(t) = \frac{mg}{k} - \frac{mg}{k}e^{-\frac{k}{m}t}$$
$$\boxed{v(t) = \frac{mg}{k}\left(1 - e^{-\frac{k}{m}t}\right)}$$

Solusi unik $v(t)$

Kecepatan Terminal :

$$v_t = v(\infty)$$
$$= \frac{mg}{k}\left(1 - \underbrace{e^{-\infty}}_{=0}\right)$$
$$= \frac{mg}{k} \Rightarrow \boxed{v_t = \frac{mg}{k}}$$

Grafik v vs t



(ii) $f = -kv^2$

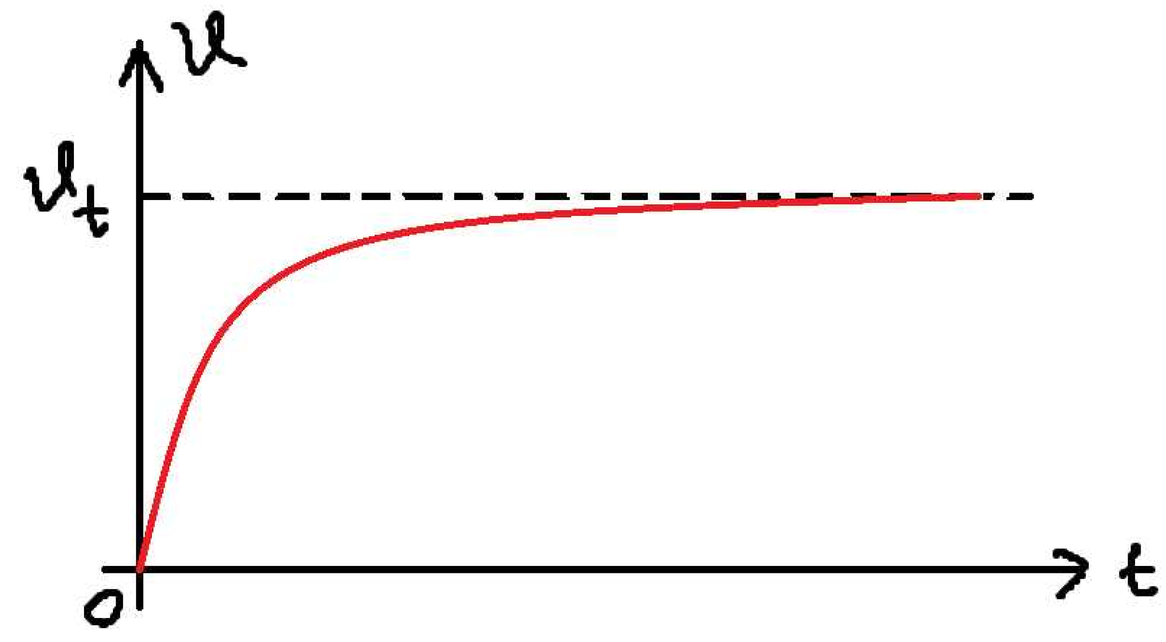
$$\Sigma F = ma$$
$$mg - kv^2 = m\frac{dv}{dt}$$
$$g - \frac{k}{m}v^2 = \frac{dv}{dt}$$
$$\frac{dv}{g - \frac{k}{m}v^2} = dt$$
$$\int \frac{dv}{\frac{mg}{k} - v^2} = \frac{k}{m}\int dt$$
$$\sqrt{\frac{k}{mg}}\tanh^{-1}\left(\sqrt{\frac{k}{mg}}v\right) = \frac{k}{m}t + C$$

$$\tanh\left(\sqrt{\frac{k}{mg}}v\right) = \sqrt{\frac{kg}{m}}t + D$$
$$\sqrt{\frac{k}{mg}}v = \tanh\left(\sqrt{\frac{kg}{m}}t + D\right)$$
$$v(t) = \sqrt{\frac{mg}{k}}\tanh\left(\sqrt{\frac{kg}{m}}t + D\right)$$
$$v(0) = 0$$
$$0 = \sqrt{\frac{mg}{k}}\tanh(D)$$
$$\tanh(D) = 0$$
$$D = 0$$
$$\boxed{v(t) = \sqrt{\frac{mg}{k}}\tanh\left(\sqrt{\frac{kg}{m}}t\right)}$$

Kecepatan Terminal :

$$v_t = v(\infty)$$
$$= \sqrt{\frac{mg}{k}}\overbrace{\tanh(\infty)}^{=1}$$
$$\boxed{v_t = \sqrt{\frac{mg}{k}}}$$

Grafik v Vs t



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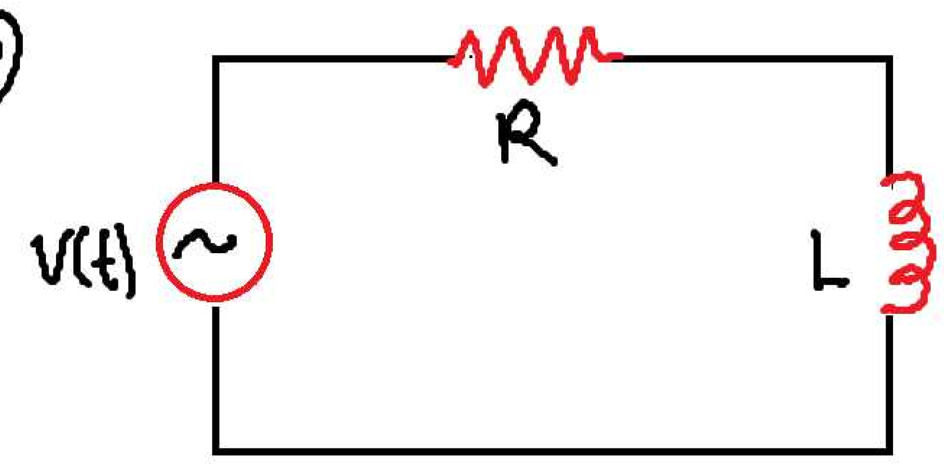
3)



$V(t) = V_0 \cos$

$\Sigma V = 0$

$iR + L\frac{di}{dt} - V_0 \cos \omega t = 0$

$\frac{di}{dt} + \frac{R}{L} i = \frac{V_0}{L} \cos \omega t = 0$

$I = \int \frac{R}{L} dt = \frac{R}{L} t$

$\mu(t) = e^{I}$

$\downarrow$

$e^{I} i = \int e^{I} \frac{V_0}{L} \cos \omega t$

$e^{\frac{R}{L}t} i = \frac{V_0}{L} \int e^{\frac{R}{L}t} \cos \omega t \quad \ldots (\bigstar)$

Menggunakan

$$\int e^{ax} \cos(bx) dx = \frac{e^{ax}(a \cos(bx) + b \sin(bx))}{a^2 + b^2} + C$$

dengan $a = \frac{R}{L}$, $b = \omega$, dan $x = t$, maka

$$\int e^{\frac{R}{L}t} \cos \omega t \, dt = \frac{e^{\frac{R}{L}t}}{\frac{R^2}{L^2} + \omega^2} \left(\frac{R}{L} \cos \omega t + \omega \sin \omega t \right) + C$$

$$e^{\frac{R}{L}t} i = \frac{V_0 e^{\frac{R}{L}t}}{\omega^2 L^2 + R^2} (R \cos \omega t + \omega L \sin \omega t) + C$$

$$\boxed{i(t) = \frac{V_0}{\omega^2 L^2 + R^2} (R \cos \omega t + \omega L \sin \omega t) + C e^{-\frac{R}{L}t}}$$

$i = \frac{dq}{dt} \Rightarrow q(t) = \int i(t) dt$

$$q(t) = \frac{V_0}{\omega^2 L^2 + R^2} \left(R \int \cos \omega t \, dt + \omega L \int \sin \omega t \, dt \right) + C \int e^{-\frac{R}{L}t}$$

$$\boxed{q(t) = \frac{V_0}{\omega^2 L^2 + R^2} \left(\frac{R}{\omega} \sin \omega t - L \cos \omega t \right) - C \frac{L}{R} e^{-\frac{R}{L}t} + D}$$

4) $p = \gamma m v$, $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$, $F = \frac{dp}{dt}$

misal : $\beta = v/c \Rightarrow p = \gamma m \left(\frac{v}{c}\right) c = \frac{m}{c} \gamma \beta$

$F = \frac{dp}{dt} = \frac{d}{dt}\left(\frac{m}{c} \gamma \beta\right)$

$F = \frac{m}{c} \frac{d}{dt}(\gamma \beta) = \frac{m}{c} \left(\dot{\gamma} \beta + \gamma \dot{\beta} \right)$

$F = \frac{m}{c} \left(\frac{\beta^2}{(1-\beta^2)^{3/2}} \dot{\beta} + \frac{1}{(1-\beta^2)^{1/2}} \dot{\beta} \right)$

$F = \frac{m}{c} \left(\frac{\beta^2 + 1 - \beta^2}{(1-\beta^2)^{3/2}} \right) \dot{\beta}$

$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$

$\dot{\beta} = \frac{d\beta}{dt}$

$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{d}{dt} (1 - \beta^2)^{-\frac{1}{2}}$

$\dot{\gamma} = -\frac{1}{2} (1 - \beta^2)^{-\frac{3}{2}} (-2\beta) \frac{d\beta}{dt}$

$\dot{\gamma} = \frac{\beta}{(1-\beta^2)^{3/2}} \dot{\beta}$

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$$F = \frac{m}{c}\left(\frac{1}{(1-\beta^2)^{3/2}}\right)\dot{\beta}$$

$$\frac{Fc}{m} = \frac{1}{(1-\beta^2)^{3/2}}\frac{d\beta}{dt}$$

$$\int \frac{d\beta}{(1-\beta^2)^{3/2}} = \frac{Fc}{m}\int dt$$

Menggunakan $\displaystyle\int \frac{1}{(1-x^2)^{3/2}}dx = \frac{x}{\sqrt{1-x^2}} + K$

maka

$$\frac{\beta}{\sqrt{1-\beta^2}} = \frac{Fc}{m}t + K$$

$$\frac{\beta^2}{1-\beta^2} = \left(\frac{Fc}{m}t + k\right)^2$$

$$\beta^2 = \left(\frac{Fc}{m}t + k\right)^2 - \left(\frac{Fc}{m}t + k\right)^2\beta^2$$

$$\beta^2\left[1 + \left(\frac{Fc}{m}t + k\right)^2\right] = \left(\frac{Fc}{m}t + k\right)^2$$

$$\beta^2 = \frac{\left(\frac{Fc}{m}t + k\right)^2}{1 + \left(\frac{Fc}{m}t + k\right)^2}$$

$$\beta = \frac{v}{c} = \frac{\left(\frac{Fc}{m}t + k\right)}{\sqrt{1 + \left(\frac{Fc}{m}t + k\right)^2}}$$

$$\Leftarrow \quad v(t) = c\,\frac{\left(\frac{Fc}{m}t + k\right)}{\sqrt{1 + \left(\frac{Fc}{m}t + k\right)^2}}$$

$\Uparrow$

$v(0) = 0$

$$\frac{kc}{\sqrt{1+k^2}} = 0 \Rightarrow k = 0$$

$$\boxed{v(t) = c\,\frac{\frac{Fc}{m}t}{\sqrt{1 + \left(\frac{Fc}{m}t\right)^2}}}$$

Saat $t \to \infty$

$$v(\infty) = \lim_{t\to\infty} c\,\frac{\frac{Fc}{m}t}{\sqrt{1 + \left(\frac{Fc}{m}t\right)^2}} = c\lim_{t\to\infty}\frac{1}{\sqrt{\underbrace{\left(\frac{m}{Fct}\right)^2}_{=0} + 1}} = c$$

$$\boxed{v(\infty) = c}$$

$$v(t) = \frac{ds}{dt} = c\,\frac{\frac{Fc}{m}t}{\sqrt{1 + \left(\frac{Fc}{m}t\right)^2}}$$

misal $u = \frac{Fc}{m}t$

$$du = \frac{Fc}{m}dt \Rightarrow c\,dt = \frac{m}{F}du$$

$$ds = \frac{u}{\sqrt{1+u^2}}\frac{m}{F}du$$

$$\frac{F}{m}\int ds = \frac{1}{2}\int\frac{d(u^2)}{\sqrt{1+u^2}}$$

$$\frac{F}{m}s = \frac{1}{2}(2)\sqrt{1+u^2} + A$$

$$s = \frac{m}{F}\sqrt{1 + \left(\frac{Fc}{m}t\right)^2} + B$$

saat $t = 0$, $s(0) = 0$

$$0 = \frac{m}{F}\sqrt{1+0} + B \Rightarrow B = -\frac{m}{F}$$

$$\boxed{s(t) = \frac{m}{F}\left(\sqrt{1 + \left(\frac{Fc}{m}t\right)^2} - 1\right)}$$

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5) $x\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy = 0$ $\quad y=\frac{1}{2} \quad x=\frac{1}{2}$

$$\int \frac{x\,dx}{\sqrt{1-x^2}} + \int \frac{y\,dy}{\sqrt{1-y^2}} = 0$$

$$\sqrt{1-x^2} + \sqrt{1-y^2} = C \Rightarrow \sqrt{1-\frac{1}{4}} + \sqrt{1-\frac{1}{4}} = C \Rightarrow C = \sqrt{3}$$

$$\boxed{\sqrt{1-x^2} + \sqrt{1-y^2} = \sqrt{3}}$$

6) $y' \sin x = y \ln y$ $\quad y = e \quad x = \pi/3$

$$\frac{dy}{dx}\sin x = y \ln y$$

$$\int \frac{dy}{y \ln y} = \int \frac{dx}{\sin x}$$

$$\ln(\ln y) = -\csc x \cot x + C$$

$$\underbrace{\ln(\underbrace{\ln e}_{1})}_{=0} = -\csc\left(\frac{\pi}{3}\right)\cot\left(\frac{\pi}{3}\right) + C$$

$$0 = -\frac{2}{3} + C \Rightarrow C = \frac{2}{3}$$

$$\Downarrow$$

$$\ln(\ln y) = -\csc x \cot x + \frac{2}{3}$$

$$\ln y = e^{-\csc x \cot x + \frac{2}{3}}$$

$$\boxed{y = e^{e^{-\csc x \cot x + \frac{2}{3}}}}$$

7) $(1+y^2)\,dx + xy\,dy = 0$ $\quad y=0 \quad x=5$

$$\int \frac{dx}{x} + \int \frac{y\,dy}{1+y^2} = 0$$

$$\ln|x| + \frac{1}{2}\ln|1+y^2| = \ln C$$

$$\ln\left|x\sqrt{1+y^2}\right| = \ln C$$

$$x\sqrt{1+y^2} = C$$

$$5\sqrt{1+0} = C \Rightarrow C = 5$$

$$\downarrow$$

$$x\sqrt{1+y^2} = 5$$

$$1+y^2 = \frac{x^2}{25} \Rightarrow \boxed{y = \pm\sqrt{\frac{x^2}{25} - 1}}$$