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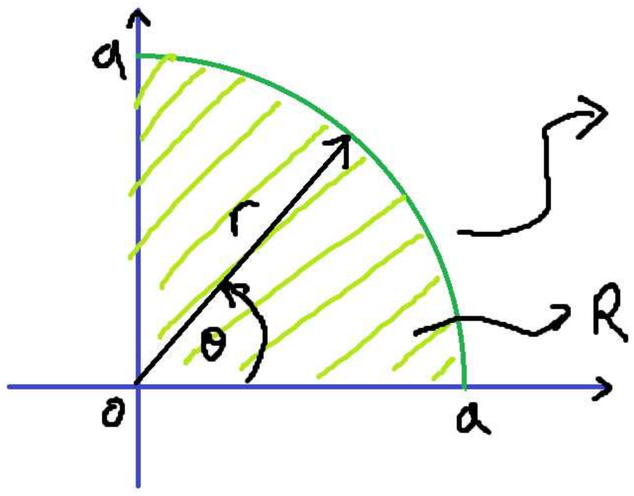
Tugas MD II : Integral Lipat Polar

(I) 1) $\int_0^a \int_0^{\sqrt{a^2-y^2}} y\,dy\,dx$

Area Domain : $x^2+y^2 \leq a^2,\ x \geq 0,\ y \geq 0$ (R)

Fungsi Z : $z = y$



$x^2+y^2 = r^2 \leq a^2 \Rightarrow 0 \leq r \leq a \qquad 0 \leq \theta \leq \pi/2$

$y = r\sin\theta \qquad dA = r\,dr\,d\theta$

$$\int_0^{\pi/2}\int_0^a (r\sin\theta)\,r\,dr\,d\theta = \int_0^{\pi/2} \sin\theta\,d\theta \int_0^a r^2\,dr = \left[-\cos\theta\right]_0^{\pi/2}\left[\frac{r^3}{3}\right]_0^a$$

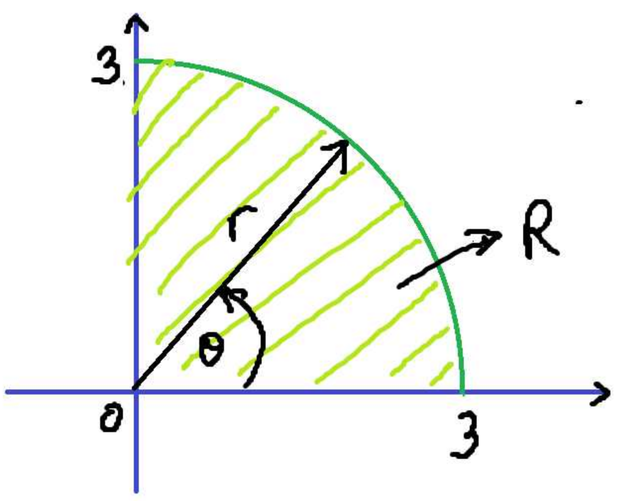
$$= (1)\left(\frac{a^3}{3}\right) = \frac{1}{3}a^3$$

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} y\,dy\,dx = \frac{a^3}{3}$$

2) $\int_0^3 \int_0^{\sqrt{9-x^2}} (x^2+y^2)^{\frac{3}{2}}\,dy\,dx$

Area Domain : $x^2+y^2 \leq 9,\ x \geq 0,\ y \geq 0$ (R)

Fungsi Z : $z = (x^2+y^2)^{3/2}$



$x^2+y^2 = r^2 \qquad dA = r\,dr\,d\theta$

$x = r\sin\theta$

$x = r\cos\theta$

$0 \leq r \leq 3$

$0 \leq \theta \leq \frac{\pi}{2}$

$$\int_0^{\pi/2}\int_0^3 (r^2)^{\frac{3}{2}}\,r\,dr\,d\theta = \int_0^{\pi/2} d\theta \int_0^3 r^4\,dr = \left[\theta\right]_0^{\pi/2}\left[\frac{r^5}{5}\right]_0^3 = \left(\frac{\pi}{2}\right)\left(\frac{243}{5}\right) = \frac{243\pi}{10}$$

$$\int_0^3 \int_0^{\sqrt{9-x^2}} (x^2+y^2)^{\frac{3}{2}}\,dy\,dx = \frac{243\pi}{10}$$

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(I.)

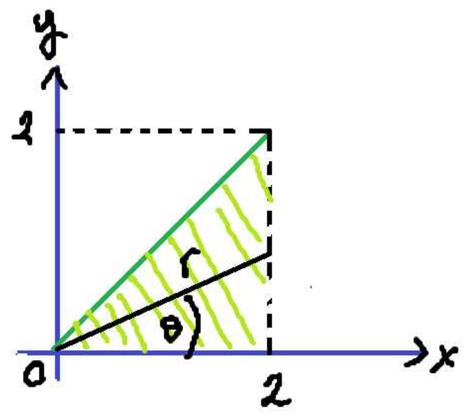
3) $\int_0^2 \int_0^x \sqrt{x^2+y^2}\, dy\, dx + \int_2^{2\sqrt{2}} \int_0^{\sqrt{8-x^2}} \sqrt{x^2+y^2}\, dy\, dx$

R = Area Domain

$\sqrt{x^2+y^2} = r$

$0 \le x \le 2$
$0 \le y \le 2$
$\rightarrow$
$0 \le r \le 2/\cos\theta$
$0 \le \theta \le \pi/4$

$x^2+y^2 \le 8$
$y \ge 0$
$x \ge 2$
$\rightarrow$
$\frac{2}{\cos\theta} \le r \le 2\sqrt{2}$
$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$



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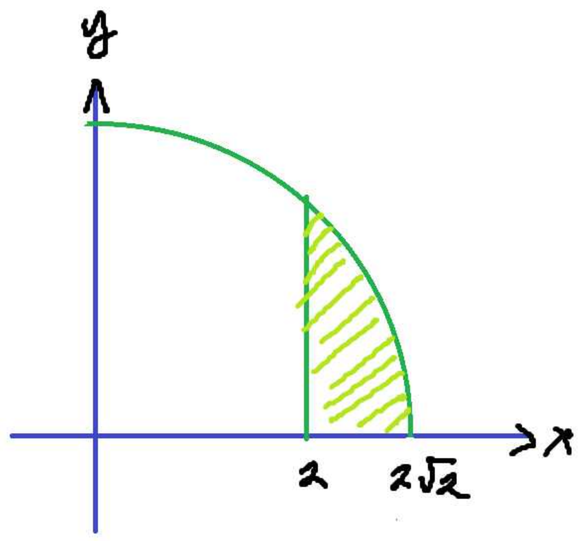

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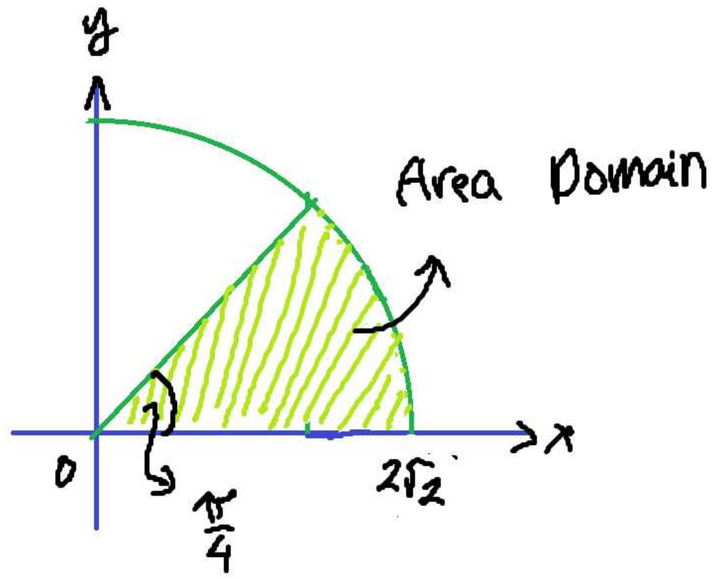

$$\int_0^2 \int_0^x \sqrt{x^2+y^2}\, dy\, dx + \int_2^{2\sqrt{2}} \int_0^{\sqrt{8-x^2}} \sqrt{x^2+y^2}\, dy\, dx = \int_0^{\pi/4} \int_0^{2/\cos\theta} r^2\, dr\, d\theta + \int_0^{\pi/4} \int_{2/\cos\theta}^{2\sqrt{2}} r^2\, dr\, d\theta$$

$$= \int_0^{\pi/4} \int_0^{2\sqrt{2}} r^2\, dr\, d\theta$$

$$= \int_0^{\pi/4} d\theta \int_0^{2\sqrt{2}} r^2\, dr$$

$$= [\theta]_0^{\pi/4} \left[\frac{r^3}{3}\right]_0^{2\sqrt{2}}$$

$$= \left(\frac{\pi}{4}\right)\left(\frac{16\sqrt{2}}{3}\right) = \frac{4}{3}\sqrt{2}\,\pi \approx 5,929$$

$$\int_0^2 \int_0^x \sqrt{x^2+y^2}\, dy\, dx + \int_2^{2\sqrt{2}} \int_0^{\sqrt{8-x^2}} \sqrt{x^2+y^2}\, dy\, dx = \frac{4}{3}\sqrt{2}\,\pi \approx 5.929$$

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(II) 4) $z = \ln(x^2+y^2)$

$z = 0,\ 1 \le x^2+y^2 \le 4 \Rightarrow$ $1 \le r \le 2$, $0 \le \theta \le 2\pi$ $\quad z = \ln(r^2)$

$$V = \int_0^{2\pi}\int_1^2 \ln(r^2)\, r\, dr\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} d\theta \int_1^2 \ln(r^2)\, d(r^2)$$

$$= \frac{1}{2}[\theta]_0^{2\pi}\left[r^2\ln(r^2) - r^2\right]_1^2$$

$$= \frac{1}{2}(2\pi)\left(4\ln(4) - 4 - 1\ln(1) + 1\right)$$

$$= 8\pi\ln(2) - 3\pi \approx 7,9959$$

$$V = 8\pi\ln(2) - 3\pi \approx 7,9959$$

5) $z = \sqrt{16-x^2-y^2}$ $\quad x^2+y^2-4x = 0 \Rightarrow (x-2)^2+y^2 = 4$

↳ z_2

Area Domain : $x^2+y^2 \le 4 \Rightarrow R \Rightarrow$ $0 \le r \le 4\cos\theta$ $\quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$$V = \iint_R z\, dA$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{4\cos\theta} \sqrt{16-r^2}\, r\, dr\, d\theta$$

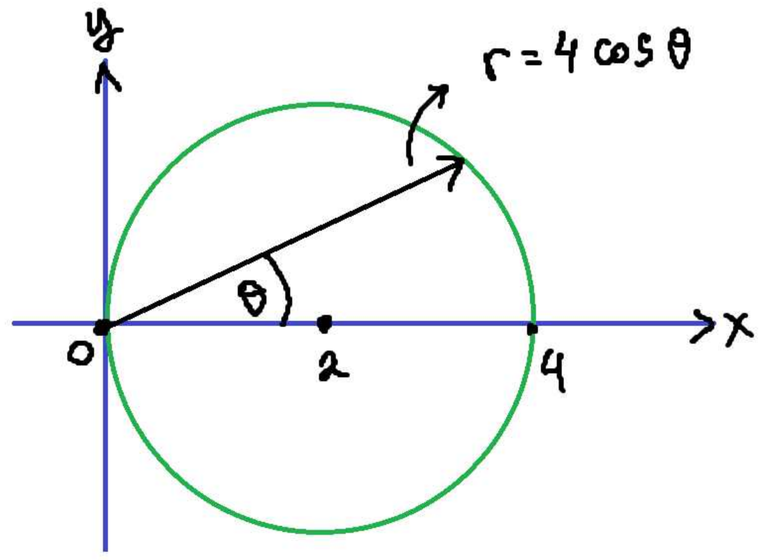
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[-\frac{1}{3}\left(16-r^2\right)^{\frac{3}{2}}\right]_0^{4\cos\theta} d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \left(\frac{64}{3} - \frac{64}{3}(\sin^2\theta)^{\frac{3}{2}}\right) d\theta$$

(fungsi genap)

$$= \frac{64}{3}\int_{-\pi/2}^{\pi/2} d\theta - \frac{128}{3}\int_0^{\pi/2} \sin^3\theta\, d\theta$$

$$= \frac{64}{3}\pi - \frac{256}{9}$$



$$\int \sqrt{a^2-x^2}\, x\, dx = -\frac{1}{3}\left(a^2-x^2\right)^{\frac{3}{2}} + C$$

$$\int \sin^3 x\, dx = \frac{1}{12}\cos 3x - \frac{3}{4}\cos x + C$$

$$\int_0^{\pi/2} \sin^3\theta\, d\theta = \frac{3}{4} - \frac{1}{12} = \frac{2}{3}$$

$$V = \frac{64}{3}(3\pi-4) \approx 38,576$$

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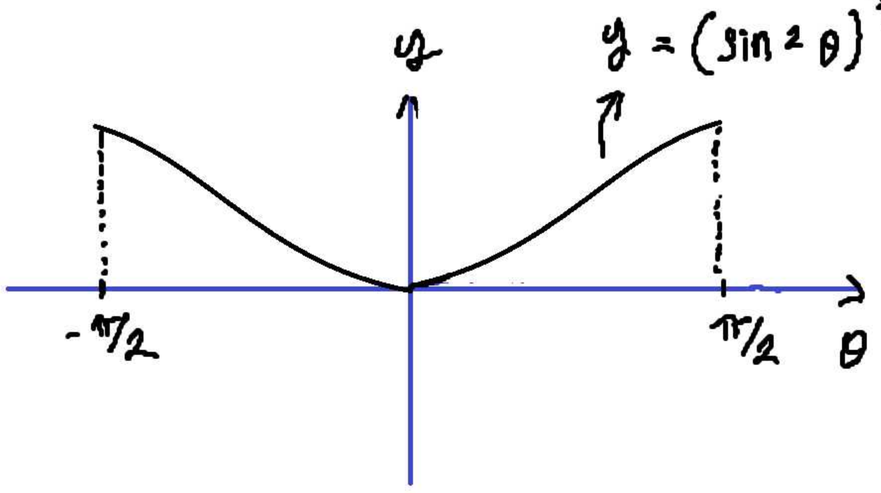
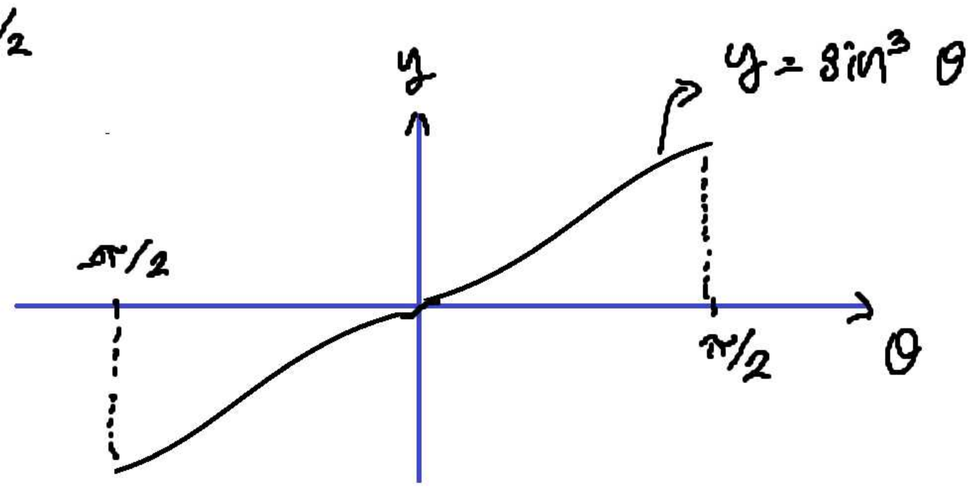
catatan untuk nomor 5 : $(\sin^2\theta)^{\frac{3}{2}} \neq \sin^3\theta$

$(\sin^2\theta)^{\frac{3}{2}}$ → Fungsi Genap

$\sin^3\theta$ → Fungsi ganjil





untuk fungsi genap berlaku $\int_{-a}^{a} (\text{fungsi genap})\, dx = 2\int_{0}^{a} (\text{fungsi genap})\, dx$

sehingga $\int_{-\frac{\pi}{2}}^{\pi/2} (\sin^2\theta)^{\frac{3}{2}}\, d\theta = 2\int_{0}^{\pi/2} (\sin^2\theta)^{\frac{3}{2}}\, d\theta$

untuk $0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq \sin\theta \leq 1$ so $(\sin^2\theta)^{\frac{3}{2}} = \sin^3\theta$

$$\int_{-\frac{\pi}{2}}^{\pi/2} (\sin^2\theta)^{\frac{3}{2}}\, d\theta = 2\int_{0}^{\pi/2} \sin^3\theta\, d\theta = \frac{4}{3}$$

atau, dengan $\theta = \frac{\pi}{2} - \phi$, $d\theta = -d\phi$

$\theta = 0 \Rightarrow \phi = \frac{\pi}{2}$

$\theta = \frac{\pi}{2} \Rightarrow \phi = 0$

$\sin\theta = \cos\phi$

$\cos\phi$ = fungsi genap

$$2\int_{0}^{\pi/2} \sin^3\theta\, d\theta = 2\int_{\pi/2}^{0} \cos^3\phi\, (-d\phi) = 2\int_{0}^{\pi/2} \cos^3\phi\, d\phi = \int_{-\pi/2}^{\pi/2} \cos^3\phi\, d\phi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3\theta\, d\theta$$

sehingga $\int_{-\frac{\pi}{2}}^{\pi/2} (\sin^2\theta)^{\frac{3}{2}}\, d\theta = \int_{-\frac{\pi}{2}}^{\pi/2} \cos^3\theta\, d\theta = \frac{4}{3}$ //