Physics Challenge for Teachers and Students

A Stubborn Skateboard

**Challenge:** A student experiments with her well-oiled skateboard by assembling the apparatus as shown. She releases the system from the state of rest altering the initial conditions. In the first experiment, the student simply releases the hanging block and the skateboard accelerates toward the pulley with acceleration $a_1$. In the second experiment, the student “locks up” the front wheels of the skateboard; once the block is released, the skateboard’s acceleration is $n$ times less than in the first experiment.

In the third experiment, the student “locks up” both the front and the back wheels of the skateboard and then releases the block. Find the ratio of the accelerations in the first and the third experiment. Consider all possible scenarios. For the “locked” wheels, assume that the coefficients of static and sliding friction are equal.

**Solution:**

![Fig. 1](image1.png) ![Fig. 2](image2.png)

Since the skateboard and the falling mass both move with the same acceleration, we can treat them as a single object. Figure 2 above shows the forces parallel to the acceleration for each of the three cases. We can apply Newton’s second law to each as follows:

- **Case 1:**
  \[ Mg = (M+m)a_1 \]  
  \[ (1) \]

- **Case 2:**
  \[ Mg - f = (M+m)a_2 \]  
  \[ (2) \]

- **Case 3:**
  \[ Mg - 2f = (M+m)a_3 \]  
  \[ (3) \]

If we solve (2) for $f$ and substitute it into (3), we get:

\[ Mg - 2(Mg - (M+m)a_2) = (M+m)a_3 \quad \text{or} \quad Mg = (M+m)(2a_2 - a_3) \]  
\[ (4) \]

Substituting $Mg$ from (1) into (4) gives:

\[ a_1 = (2a_2 - a_3) \]  
\[ (5) \]

Now, the problem stated that the acceleration in Case 2 was $n$ times less than in Case 1, so $a_2 = a_1/n$ and therefore

\[ \frac{a_1}{a_3} = \frac{2}{2-n} \]

Clearly, this only makes sense for $n < 2$. If $n \geq 2$, then there is sufficient friction to keep the cart from moving at all. Rather than write the ratio of $a_1$ to $a_3$, it might be better to look at the ratio of $a_3$ to $a_1$. Then:

\[ \frac{a_3}{a_1} = \begin{cases} \frac{2-n}{n} & 1 \leq n < 2 \\ 0 & n \geq 2 \end{cases} \]

**Contributed by H. Scott Wiley, Science Academy of South Texas, Weslaco, TX**

Many other readers also sent us the correct solutions. We would like to recognize the following contributors:

Phil Cahill (Lockheed Martin Corporation, Rosemont, PA)
Fernando Ferreira (Universidade da Beira Interior, Covilhã, Portugal)
Bruce Gordon (Kimball Union Academy, Meriden, NH)
Art Hovey (Amity Regional High School, Woodbridge, CT)
Carl E. Mungan (U. S. Naval Academy, Annapolis, MD)
Gregory Ruffa (University of Minnesota, Minneapolis, MN)
Leo H. van den Raadt (Heemstede, The Netherlands)

Many thanks to all contributors and we hope to hear from you in the future!