Satel-Light Motion

On planet Tehar, the index of refraction of the atmosphere depends on the altitude as $n(h) = n_0 - bh$, where $b$ is a constant coefficient ($b \ll n_0/h$). A Teharian scientist shines a laser beam horizontally from the top of the highest mountain on Tehar. The scientist is surprised to discover that the laser beam orbits the planet and hits the back of his head. How high is the mountain? The radius of the planet is $R$.

Solution: Let’s suppose the atmosphere on Tehar is composed of many thin layers of gas with thickness $\Delta h$ each, as is shown in the figure, and that the refraction index is constant in each layer. That is, the layer that begins at the height $h$ has a refraction index $n(h)$, and the one that begins at $h + \Delta h$ has a refraction index $n(h + \Delta h)$. If the height $h$ is sufficient, the laser beam will undergo total internal reflection in the interface with the upper layer, because the critical angle $\theta$ is reached. Using the Snell law we have

$$n(h) \sin \theta = n(h + \Delta h).$$  (1)

On the other hand, from the figure, and taking into account that $\Delta h \ll R + h$, we can see that

$$\sin \theta = \frac{R + h}{R + h + \Delta h} = \frac{1}{1 + \frac{\Delta h}{R + h}} \approx 1 - \frac{\Delta h}{R + h}.  \quad (2)$$

Substituting Eq. (2) in (1) and doing a little algebra, we get the equation:

$$\frac{n(h + \Delta h) - n(h)}{\Delta h} = \frac{n(h)}{R + h}.  \quad (3)$$

If we set $\Delta h \to 0$, Eq. (3) becomes the following differential equation:

$$\frac{dn}{dh} = - \frac{n(h)}{R + h}. \quad (4)$$

But, as we know, on Tehar $n(h) = n_0 - bh$, so $\frac{dn}{dh} = -b$.

Substituting this into Eq. (4), we obtain:

$$h = \frac{n_0 - bR}{2b}.$$

(Contributed by Rodrigo Pelayo, UPIITA-Instituto Politécnico Nacional, México, D.F., México)

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Marianne Breinig (The University of Tennessee, Knoxville, TN)
Phil Cahill (Lockheed Martin Corporation, Rosemont, PA)
Don Easton (Lacombe, Alberta, Canada)
John F. Goehl, Jr. (Barry University, Miami Shores, FL)
Fredrick P. Gram (Cuyahoga Community College, Cleveland, OH)
Art Hovey (Milford, CT)
J. Iñiguez (Universidad de Salamanca, Salamanca, Spain)
Stephen McAndrew (Trinity Grammar School, Summer Hill, NSW, Australia)
Carl E. Mungan (U.S. Naval Academy, Annapolis, MD)
Leo H. van den Raadt (Heemstede, The Netherlands)

Many thanks to all contributors and we hope to hear from you in the future!
Please send correspondence to:

Boris Korsunsky
444 Wellesley St.
Weston, MA 02493-2631
korsunbo@post.harvard.edu