**Physics Challenge for Teachers and Students**

**Solutions to March 2007 Challenge**

**No Honey Here**

**Challenge:** The diagram shows a part of an infinite circuit made of conducting wire. Each side of each hexagon has the same resistance $R$ (unknown). The ohmmeter connected to points K and L reads 10 $\Omega$. Find $R$.

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**Solution:** A method relying on symmetry and superposition to find the effective resistance between adjacent nodes on an infinite square mesh of resistors was given by Aitchison. We will begin by reviewing that method as applied to the adjacent node problem on the infinite hexagonal mesh. Then we will obtain the effective resistance between a pair of nodes, such as K and L, which have one node in between them.

Adjacent nodes K and M are held at a potential difference $V$. This physical situation is viewed as a superposition of two other configurations. In one configuration the point K is held at potential $V/2$ relative to the boundary at infinity. A current $i$ flows into K and splits evenly three ways owing to symmetry of the mesh and to Kirchhoff’s junction rule. In particular, a current $i/3$ flows from K to M. The superposition of these two configurations gives a net current $2i/3$ in the branch KM and a potential difference $V$ between its ends. By Ohm’s law the potential drop through the resistance in KM is $(2i/3) R$.

By Kirchhoff’s loop rule, we have

$$V = \frac{2i}{3} R,$$

and the effective resistance is therefore given by

$$R_{KM} = \frac{V}{i} = \frac{2}{3} R.$$

The effective resistance between K and L is found by the same technique. If K is held at potential $V/2$, the current $i$ splits three ways. The current $i/3$ in KM then splits two ways so that the current in ML is $i/6$. By holding L at $-V/2$ we introduce a current of $i/3$ in ML (from M toward L) and $i/6$ in KM. Superposition yields current $i/3 + i/6 = i/2$ in KM and the same in ML. We find

$$V = i_{KM}R + i_{ML}R = iR$$

and the effective resistance is

$$R_{KL} = R = 10 \ \Omega.$$

**Note:** The effective resistance between adjacent nodes on an infinite square mesh is easily calculated to be $R/2$, and similarly it is found to be $R/3$ on an infinite triangular mesh. The symmetry argument, however, breaks down for alternating nodes, requiring a different
approach for the square and triangular cases.\textsuperscript{3,4}


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