# Objects with negligible mass

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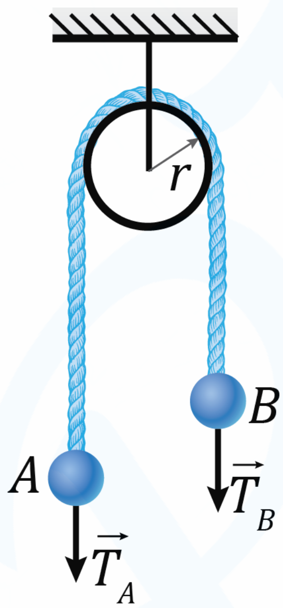
- With any suggestions please write to **feedback@physolymp.com**

One particular interesting type of problems deals with elements of the system with negligible mass. For such cases a system usually behaves in a special way, honoring fundamental laws and at the same time prohibiting infinitely large accelerations, which could incur for elements with a mass close to zero

Let's consider a well known example of the weightless pulley, non-stretchable weightless rope and two bodies $A$ and $B$ attached to the rope as shown at the picture below:




If there is no slippage between the rope and pulley and no friction in the axle of the block, equation of rotational dynamics for the pulley with moment of inertia $I$ can be written as

$$I\varepsilon = T_B r - T_A r$$

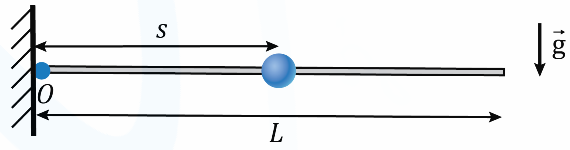
However, in this example is assumed that mass of the pulley is close to zero, which means that its moment of inertia is also almost zero. This corresponds to the case of infinitely large angular acceleration $\varepsilon$ if tension forces from both sides of the pulley are not equal. So, to honor both fundamental equation of rotational dynamics and have physically meaningful kinematic characteristics of the system, tension force in the string should be equal at both sides of the block:

$$T_A = T_B$$

Finding similar relationships or other specific parameters of the system inherited to the elements with negligible mass is quite challenging, but rewarding, usually with unexpectedly simple answers

**Example 1**

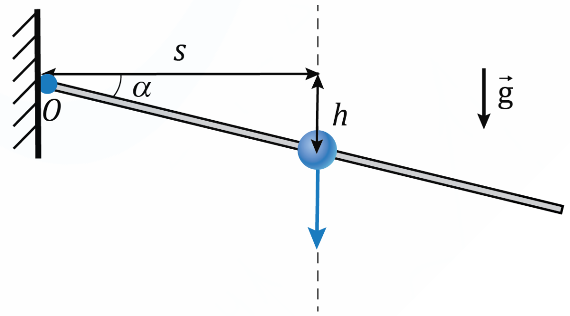
A thin rigid rod can rotate without friction around one of its ends attached to a hinge $O$. The rod goes through a heavy bead, which is placed initially at the distance $s$ from the joint $O$. Find angle $\alpha$, which the rod makes with a horizontal line after a period of time $t$. Assume that initial position of the rod is horizontal, while length of the rod $L$ is large enough, so the bead is still on the rod after time $t$




As mass of the rod $m$ is close to zero, any non-zero reaction force $\vec{N}$ from the bead on the rod would lead to infinitely large angular acceleration $\varepsilon$ of the weightless rod

$$\varepsilon \sim \frac{\vec{N}}{m}$$

So, for physically meaningful values of acceleration of the rod, reaction force should be zero, which corresponds to the case of free fall of the bead in the gravity field $\vec{g}$




After a period of time $t$, the bead goes down by distance $h$

$$h = \frac{gt^2}{2}$$

If separation between the bead and hinge is less than length $L$ of the rod, angle $\alpha$ can be expressed with a trigonometric relationship

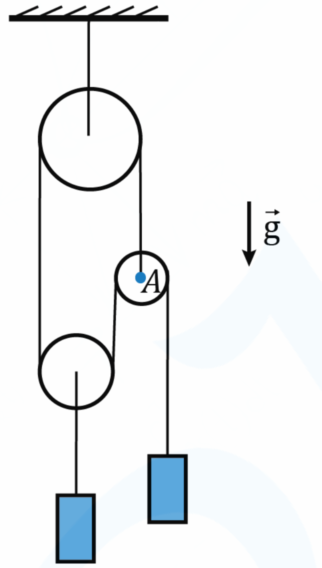
$$\sin\alpha = \frac{h}{s}$$

Thus,

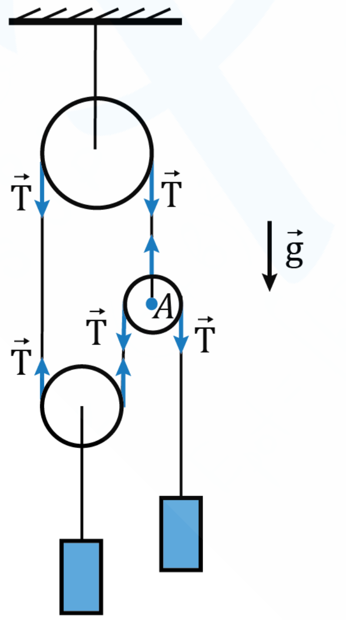
$$\boxed{\alpha = \arcsin\left(\frac{gt^2}{2s}\right)}$$

**Example 2**

A system consists of two bodies attached to the non-stretchable weightless ropes through three pulleys as shown at the picture. Masses of all pulleys are close to zero, there is no friction or slippage in the system. Find acceleration $a_A$ of the pulley $A$ in the gravity field $g$




As the rope is weightless, tension force $T$ should be equal in any point of the rope.



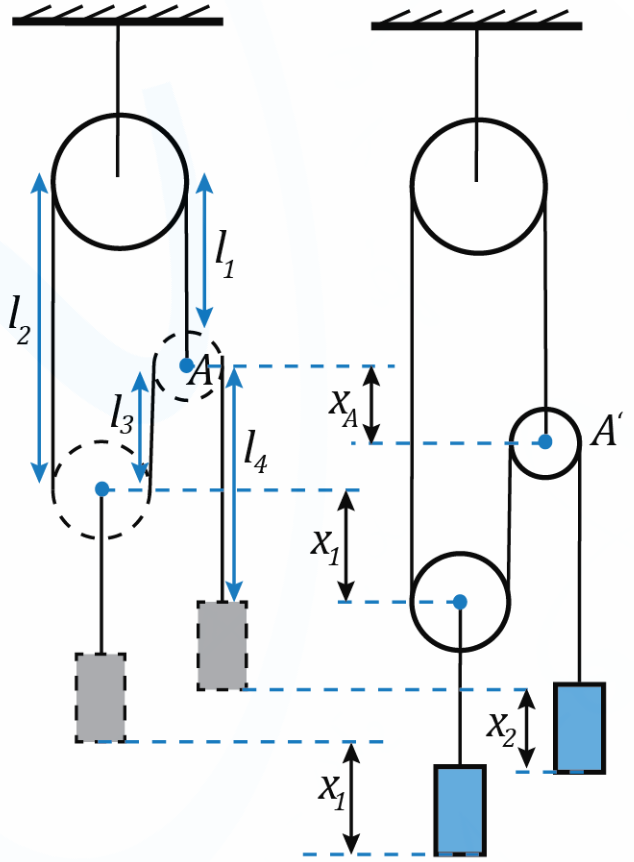

Newton's law for the pulley $A$ with a mass $m_A \approx 0$ can be written as

$$a_A = \frac{2T - T}{m_A} = \frac{T}{m_A} \to \infty \quad if \quad T \neq 0$$

Thus, the only physically meaningful solution for this system corresponds to zero tension force. So, both of the weights will drop freely in the gravity field

$$a_1 = a_2 = g$$

Let's find a kinematic relationship between displacement $x_A$ of the pulley $A$ and displacements of the weights $x_1$ and $x_2$ by examining difference between snapshots of the system taken in small time interval:




For the case of non-stretchable rope, total length of the string should be constant for any moment of time:

$$l_1 + l_2 + l_3 + l_4 = (l_1 + x_A) + (l_2 + x_1)$$
$$+(l_3 - x_A + x_1) + (l_4 - x_A + x_2)$$

Rearranging variables yields

$$x_A = 2x_1 + x_2$$

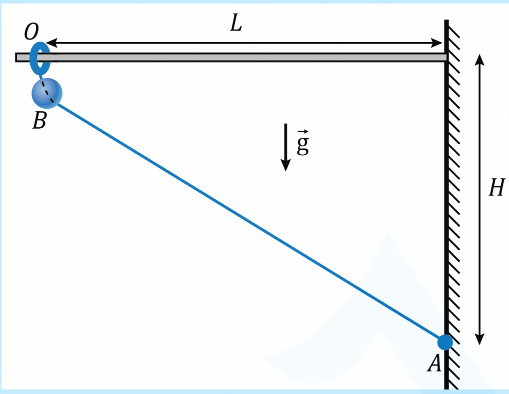
Differentiating two times gives

$$\ddot{x}_A = 2\ddot{x}_1 + \ddot{x}_2 = 2g + g$$

$$\boxed{a_A = 3g}$$

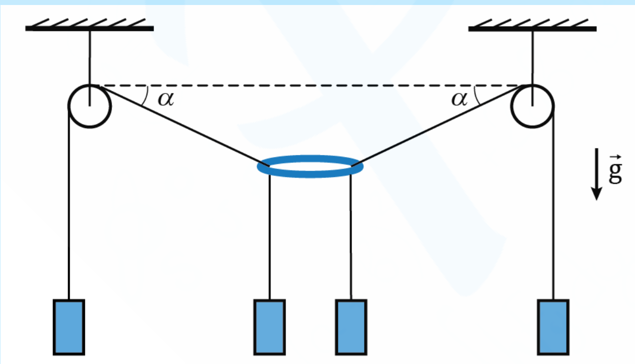
## Problem 1

One end of the lightweight non-stretchable rope is fixed to the wall at the point $A$, while another end of the rope is fastened to the weightless small ring. The ring can move without friction along horizontal bar welded to the wall at the distance $H = 1.0\ m$ above the point $A$ as shown at the picture below. A string goes through a heavy bead $B$, which can slide along the thread without friction. Initially, the bead rests near the ring, which was located at the maximum distance $L = 2.0\ m$ from the wall. Find displacement $s$ of the ring from initial position to the moment, when the bead will lower by height $h = 0.5\ m$ descending in the gravity field



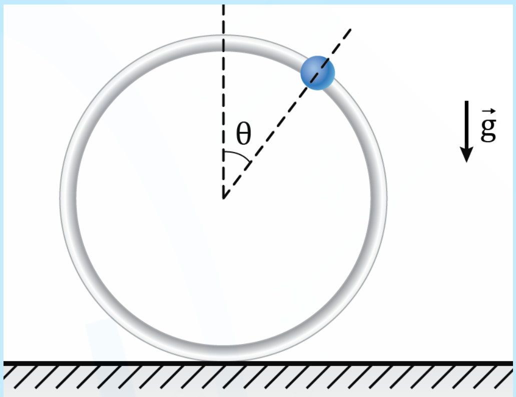

## Problem 2

Two pairs of identical weights are connected with massless non-stretchable strings. Those strings go through a weightless ring as shown at the picture below. Both pulleys have negligible mass and rotate without friction. Entire system is under influence of gravity field $g = 9.8\ m/s^2$. Find initial acceleration $a_0$ of the ring, when the system got free. Initial angle is $\alpha = 30°$



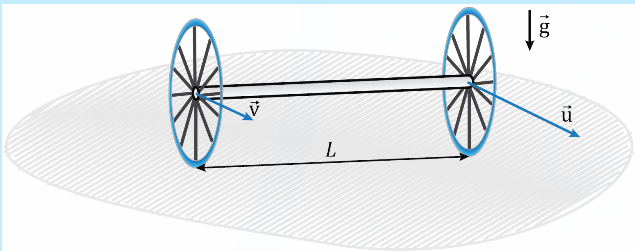

## Problem 3

A small heavy bead is attached to the ring, which has negligibly small mass. The ring is placed at the horizontal surface with a coefficient of friction $\mu = 0.1$. Initially, the bead is located at the highest point of the ring. Find angle $\theta$ with vertical as shown at the picture, corresponding to the moment, when the ring will start slipping along the horizontal surface




## Problem 4

Two small wheels with negligible mass are attached to the heavy rod of a length $L = 1.0\ m$. This system is placed at the flat horizontal surface with a coefficient of friction $\mu = 0.1$. Both of the wheels are pushed in the direction perpendicular to the axis and parallel to the horizontal surface as shown at the picture below:




Magnitude of initial velocity for one of the wheels is $v = 1.0\ m/s$. What is maximum possible velocity of the second wheel $u$ for a motion of the system without slippage along the flat surface in the gravity field $g = 9.8\ m/s^2$?