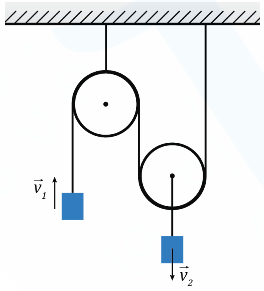
Kinematics of non-stretchable ropes

- Answers and detailed solutions to all problems are provided in iOS/Android "**PhysOlymp**" app
- With any suggestions please write to **feedback@physolymp.com**

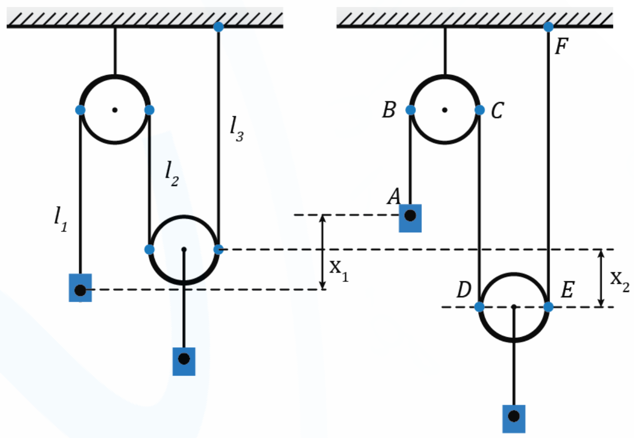
The first application of physics that we are going to cover is related to complex moving structures, which can be described with trivial algebraic equations. In this chapter is discussed only kinematic relations for parts of the system connected with non-stretchable ropes via pulleys. To solve such problems, one needs to look at the system in a small period of time and describe total length of the rope in the system as a constant parameter

Example 1

A polyspast consists of two pulleys, with two bodies hanged at non-stretchable ropes as shown at the picture. What is velocity of the second body v_2 if velocity of the first object is v_1?



The most important part is to make a good sketch with system parallel to itself in a small period of time Δt



After that, writing equations is very easy

$$l_1 + l_2 + l_3 = AB + CD + FE$$

where was accounted that for non-stretchable rope, its total length at any moment of time should be constant. The most convenient way to obtain solution is usually by expressing length of the rope in a little increments x_i after a short period of time Δt, as

$$l_1 + l_2 + l_3 = (l_1 - x_1) + (l_2 + x_2) + (l_3 + x_2)$$

All terms for initial length of the parts of the rope canceling out, with only following variables left

$$x_1 = 2x_2$$

At the same time, those small displacements can be expressed as a distance covered with constant speed during short period of time

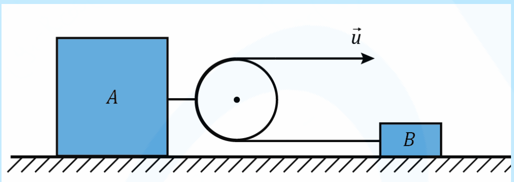
$$v_1 \Delta t = 2 v_2 \Delta t$$

Finally, will get a following relation

$$\boxed{v_2 = \frac{v_1}{2}}$$

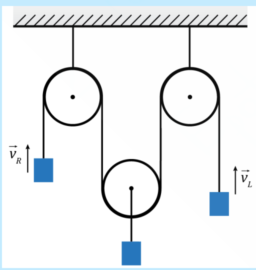
Problem 1

Two bodies A and B are connected with non-stretchable rope via pulley as shown at the picture below. How fast should be moved a free end of the rope, characterized with velocity u, so both of the bodies would approach with equal velocities $v_A = v_B = 1.0\ m/s$?



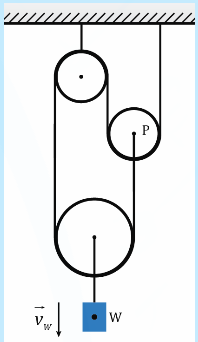
Problem 2

The system consists of the three heavy objects connected with ropes and pulleys as shown in the picture. What is velocity of the middle body v_M, if left and right objects are moving upward with velocities $v_R = 1.0\ m/s$ and $v_L = 3.0\ m/s$ respectively?



Problem 3

For the system shown at the figure below, it is known, that weight W moves downwards with constant velocity $v_w = 1.0\ m/s$. Assuming that ropes are non-stretchable, find velocity v_p of the pulley P

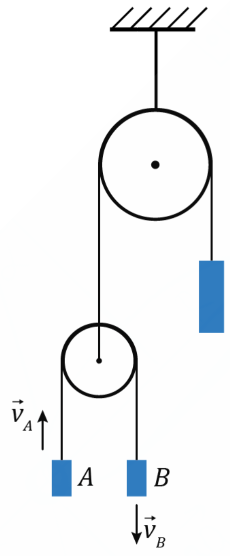


In more complex systems can be encountered several separate non-stretchable strings connected to the weights and pulleys. For kinematic description of the system, is required a similar approach of drawing parallel snapshots in a small time intervals should, but with writing "length conservation" laws for each of the non-stretchable ropes in the system. It is advised to be consistent, by starting at one end of the string and adding its sections consequitively, one by one, without changing the order till the end of the rope. Consistency helps elimination of errors related to skipping some of the section in a complex system or counting it twice

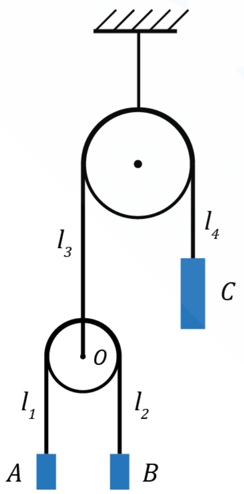
Example 2

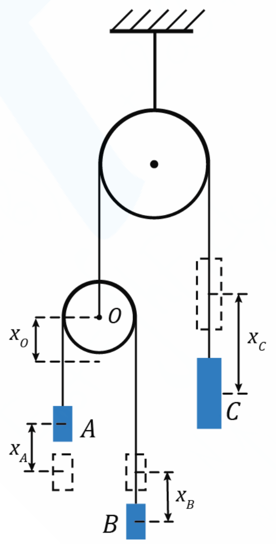
Atwood machine consists of two pulleys and three weights A, B and C connected with non-stretchable strings as shown at the picture below. Find v_C - velocity of the object C at the moment, when velocities of the weights A and B are v_A and

v_B respectively



The only difference with previously discussed problems is the fact that there are two separate strings in the system, not one. Thus, equations of constant length of the rope should be written for both of the ropes: one connecting objects A and B and another rope thrown over upper, fixed pulley connecting object C with a movable block O





Let's denote initial lengths of the vertical sections of the ropes as l_i, displacements of the object as x_i, which occurred after a small time interval Δt. Equation of constant length for the lower rope AOB can be expressed as

$$l_1 + l_2 = (l_1 + x_O - x_A) + (l_2 + x_O + x_B)$$

simplifying leads to

$$2x_O - x_A + x_B = 0 \tag{1}$$

Similar approach for the upper string OC:

$$l_3 + l_4 = (l_3 - x_O) + (l_4 + x_C)$$

or,

$$x_O = x_C \tag{2}$$

Eliminating variable x_O from equations (1) and (2) gives

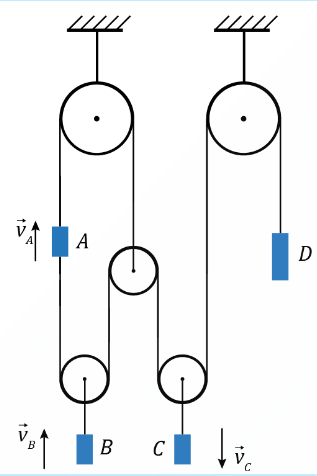
$$2x_C - x_A + x_B = 0$$

Using relation between velocity v_i of the i^{th} object with its displacement x_i as $x_i = v_i \Delta t$ results in a final answer:

$$\boxed{v_C = \frac{v_A - v_B}{2}}$$

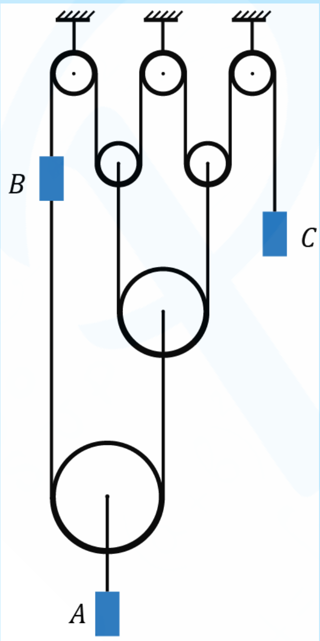
Problem 4

Consider a system consisting of several pulleys, non-stretchable massless strings and four moving bodies A, B, C and D, as shown at the picture. At some moment of time, velocities of the weights A, B and C are measured as $v_A = 0.5\ m/s$, $v_B = 1.0\ m/s$ and $v_C = 2.0\ m/s$ respectively. Find v_D - velocity of the object D at that moment



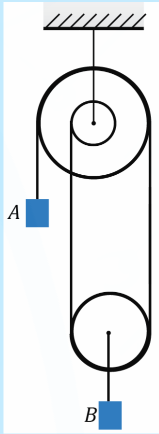
Problem 5

Three weights A, B and C are connected with a non-stretchable ropes via a system of pulleys as shown at the picture below. What is v_C - velocity of the body C, at the moment, when other objects A and B descend with velocities $v_A = 1.0\ m/s$ and $v_B = 1.0\ m/s$ respectively?



Problem 6

The system consists of weights A and B, non-stretchable ropes, movable pulley connected to the lower body B and another pulley in form of a spool as shown at the picture below



What is velocity of the lower weight v_B, if upper body moves with constant velocity $v_A = 1.0\ m/s$? Assume that rope is not slipping while pulleys are rotating, and inner radius of the spool has half size of the outer radius